

Spoor A2:

Estimating potential employment and NAIRU for Belgian regions.

A. Sabadash

KULeuven CES Juli 2009

Algemeen secretariaat – Steunpunt beleidsrelevant Onderzoek Fiscaliteit & Begroting Voskenslaan 270 – 9000 Gent – België Tel: 0032 (0)9 248 88 35 – E-mail: vanessa.bombeeck@hogent.be www.steunpuntfb.be

Estimating potential employment and NAIRU for Belgian regions.

Anna Sabadash CES - Monetary and Information Economics, KU Leuven

July 2009

1 Introduction

The value of the non-accelerating inflation rate of unemployment (NAIRU) for policy questions keeps on gaining its importance in the last two decades. Setting an economic policy requires, among others, an identification of the sustainable rate of capacity utilization. In this sense sustainability is normally associated with reasonably stable inflation. Thus, in terms of labor market the idea of sustainable resources utilization is closely connected to the concept of potential employment and the NAIRU which provides a benchmark to identify sustainable and unsustainable trends in unemployment and output. Short term employment-stimulating economic policies alone can not permanently effect equilibrium unemployment: the NAIRU may, to some degree, be influenced by the path of actual employment, but its shifts are mainly determined by structural factors. Temporary supply shocks may alter the rate of inflation, while the NAIRU will be basically unaffected once they are outdone; by contrast, long-term supply shocks (caused, for example, by demographic changes) may permanently modify the NAIRU path.

Despite the prominence of the NAIRU concept, its empirical implementation often provides controversial results since the variable itself is unobservable and not always well defined. As a result, alternative methodologies for measuring the NAIRU can lead to important divergences in their estimates. Country administrations and international economic authorities, however, find it useful to apply NAIRU estimates as an input to various assessments of fiscal and structural policies. Model specification and choice of estimation method become an important issue in the given context.

This paper first provides a review of alternative theoretical approaches broadly classified into three groups: statistical, structural and combined methods. Conceptual basis of different assessments and their ability to provide precise estimates are analyzed with respect to the usefulness for economic policy analysis. Since the conventional wisdom and officially adopted approach of the leading international economic organizations for estimating the NAIRU relies on the existence of an expectations-augmented Phillips-type relationship, further in this paper it is suggested to model potential labor output using two structural equations: a wage setting equation in the form of an expectation augmented Phillips curve and an employment equation allowing for supply shocks as proxied by the labor force participation rate and commuting between regions. The NAIRU is estimated from the modified Phillips curve equation ("triangle" Phillips model) using the Kalman filter.

The reminder of this paper is organized as follows. Section II contains a review of existing NAIRU estimation technics. Section III presents estimation procedure, including the empirical framework, data description and estimation results. The final section concludes.

2 Review of Existing Techniques

The NAIRU depends on a broad variety of economic and institutional factors and, being an unobserved variable, it can only be estimated with uncertainty. A wide range of methods have been developed for this purpose. Here we suggest a short review of the most commonly used techniques and discuss their advantages and pitfalls. While it should be admitted that methods classification is rather questionable three groups are usually made out from the variety of approaches¹:

- statistical (or time-series)
- economical (or structural)
- combined (compromise between the previous two)

Although the approaches differ, they all aim at removal of business-cycle fluctuations effects from the observed time series for unemployment.

2.1 Statistical Methods

Trending Methods.

Trending methods suggest to decompose actual unemployment into a deterministic trend component and a cyclical component, with the former identified as the NAIRU. The trend component of unemployment is supposed to be a linear function of time and is estimated by means of a linear regression of the log unemployment rate on a constant and a time trend.

$$\ln U_t = \alpha + \beta t + e_t \tag{1}$$

The NAIRU in this equation is given by $(\alpha + \beta t)$, while potential unemployment growth rate is estimated by the slope and is supposed to be constant.

Among the most cited pitfalls of using this estimation technique are the following three: trending can not allow for any supply shock; the constant growth rate of potential unemployment is assumed; the cyclical unemployment can be biased by partially allocating trend components into the cyclical component (because the stochastic trend is not fully eliminated when the resulting gaps are not stationary) (Cotis *et al.*, 2005)

¹There are still some overlaps in such classification, see (Cotis *et al.*, 2005)

Univariate filters

The objective of filtering is to update our knowledge about the system each time a new observation is brought in (filtering = detrending).

a) Hodrick Prescott filter

Filtering is performed by introducing a trade off between a good fit to the actual series (1st term in equation (2)) and the degree of smoothness of the trend series (2nd term in equation (2)).

$$HP = \min \left\{ \begin{array}{c} \sum_{t=1}^{T} \left(\ln U_t - \ln U_t^* \right)^2 + \\ +\lambda \sum_{t=1}^{T} \left(\left[\ln U_{t+1}^* - \ln U_t^* \right] - \left[\ln U_t^* - \ln U_{t-1}^* \right] \right)^2 \end{array} \right\}$$
(2)

where U_t and U_t^* are actual unemployment and trend unemployment (or NAIRU) respectively and λ is Lagrange multiplier.

The method has been criticized mostly on two grounds: for imprecise estimates of the time series end points and an absence of a common rule for choosing an objectively correct value for the parameter λ . Besides, the Hodrick Prescott (HP) filter can not capture structural breaks in the trends of economic series. One of the most commonly used improvements of the HP filter is an extension of the forecast period over the data sample in order to mitigate the end-of-sample problem.

b) Watson-Clark unobserved components model

The unobserved component (UC) model was developed by Watson (Watson, 1986) and Clark (Clark, 1987) and assumes that macroeconomic time series contain trend, cycle and, in some cases, erratic components which are not directly observable. Decomposition into these three elements can be performed by imposing sufficient restrictions on the trend and the cycle. For example, the log of observed unemployment is assumed to be:

$$u_t = u_t^p + c_t \tag{3}$$

The first term stands for the permanent component and the second for the cyclical one. The two components are not correlated with each other.

The *permanent component* can be seen as an estimate of the NAIRU and is often specified as

$$u_t^p = u_{t-1}^p + \mu_{t-1} + \eta_t \tag{4}$$

$$\mu_t = \mu_{t-1} + \varsigma_t \tag{5}$$

with η_t and ς_t orthogonal white noise.

The transitory cyclical component is unobserved stationary AR(2):

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \gamma_t \tag{6}$$

where γ_t is white noise and ϕ is an unknown parameter which in classical analysis is replaced by its maximum likelihood estimate.²

 $^{^{2}}$ In terms of state space model (see later in the Section): equation (3) is an observation equation, equations (4)-(5) are transition equations.

UC model is usually estimated by the Kalman filter, which has two distinct phases: predict and update. The predict phase uses the state estimate from the previous timestep to produce an estimate of the state at the current timestep. In the update phase, measurement information at the current timestep is used to refine this prediction to arrive at a new, (hopefully) more accurate state estimate, again for the current timestep.

Univariate filters have been questioned for their ability to appropriately distinguish between the permanent and transitory components of the time series.

The basic problem with all statistical methods is that they do not involve economic priors and depend on arbitrary assumptions in order to perform decomposition into trend and cyclical components. For example, in the case of a HP filter, trend unemployment is identified as a weighted moving average of actual unemployment, while in a Kalman Filter it is assumed to be a random walk. Another inconvenience is due to the fact that the indicators obtained are not well defined since all the information other then observed unemployment is ignored and leads to the trend estimates which are usually centered around actual time series. Besides, another serious drawback is the difficulty to judge the degree of precision of the result in most cases.

2.2 Structural Methods

Structural methods attempt to estimate the NAIRU by means of aggregate structural models of wage and price setting behavior. These approaches require a strong assumption of full or partial market equilibrium where wages are bargained between workers and firms and the latter decide on the level of employment, prices and output after a wage agreement has been reached. Both employers and firms have some market power, since hiring or firing workers is costly for both parties. Product market conditions, the capital stock and technology are assumed to be exogenously given.

An example of a structural model is presented bellow. This model suggests to estimate a system of equations describing wage, price settings and labor supply as follows (Richardson *et al.*, 2000).

Price equation:

$$p - w = \alpha_0 + \alpha_1 n + \alpha_2 \Delta n + \alpha_3 (p - p^e) - le + LT^p + ST^p \tag{7}$$

Wage equation:

$$w - p = \beta_0 - \beta_1 U - \beta_2 \Delta U - \beta_3 (w - w^e) + le + LT^w + ST^w$$
(8)

Labour supply

$$l = \gamma_0 - \gamma_1 U + LT^l \tag{9}$$

where U is unemployment, p, w, n, l and le are respectively prices, wages, employment, labor force and trend labor efficiency in the logarithmic form, Δ is the first difference operator, and LT and ST are vectors of exogenous variables which represent long-term and short-term shocks.³

The long-term equilibrium unemployment can be found from solving the system of price, wage and labor supply equations, assuming that in the long run price and wage expectations are met $((w - w^e) = (p - p^e) = 0)$, unemployment rate does not change (ΔU) , short-term shocks disappear and the long-term shocks adjust to their long-term equilibria values. This definition of the NAIRU implies its responsiveness to the changes in the main institutional characteristics of the labor and product markets.

Another example of structural models is a structural autoregressive model (SVAR). This model was developed by Blanchard and Quah (Blanchard & Quah, 1989). The method is based on a structural autoregressive model that estimates potential output and the output gap using structural assumptions about the nature of economic disturbances. The model uses the information about employment and capacity utilization to decompose actual time series into a permanent trend component (supply) and temporary cyclical component (demand). Each of the three variables is regressed in the system on their own lags and the lags of the other variables. The shocks are grouped into supply and demand shocks. The key assumptions of the model are that demand shocks do not affect output in the long run whereas supply shocks do. The variables included in the different applications differ but in every case the main objective is to remove fluctuations due to demand conditions from the output series in order to determine potential output. Potential output is calculated by historical decomposition addressing the question: what would the output series look like in the absence of demand shocks.

Structural models provide a robust theoretical framework to explain the impact of different supply and demand shock as well as policy instruments, but at the same time fail to produce precise NAIRU estimates. A number of empirical studies used this approach to estimate the output gap by removing fluctuations due to demand and supply shocks from the output series. For example, Funke (Funke, 1997) proxied demand shocks by manufacturing output and inflation and calculated potential output for Germany by historical decomposition answering the question: what would the output series look like in the absence of demand shocks? In Hjelm (Hjelm, 2003) a structural VAR approach is used to estimate the Swedish NAIRU, output gaps and structural budget balances in the same model. The author concludes that; despite its several weaknesses (long data series requirement and additional data adjustments like, for instance, the integration tests) this method could be useful as a complement to the conventional estimation techniques used by the OECD, EC and other research institutes.

2.3 Combined Methods

Multivariate filters

³A good assessment of the model can be find in (Chiarinia & Piselli, 2000).

In response to the criticism of the limitations of the univariate methods, a variety of multivariate extensions of univariate filters have been proposed. Thus, multivariate HP (MVHP) was suggested by Laxton and Teltow (Laxton & Tetlow, 1992) who conditioned the HP filter estimate of cyclical component on additional relevant information. Practically, the residuals of price Phillips curve or/and an Okun's relationship ($\varepsilon_{\pi,t}^2$, $\varepsilon_{U,t}^2$) are added to the classical HP specification (2)(Cotis *et al.*, 2005):

$$HP = \min \left\{ \begin{array}{c} \sum_{t=1}^{T} \left(\ln U_t - \ln U_t^* \right)^2 + \\ +\lambda \sum_{t=1}^{T} \left(\left[\ln U_{t+1}^* - \ln U_t^* \right] - \left[\ln U_t^* - \ln U_{t-1}^* \right] \right)^2 + \\ +\sum_{t=1}^{T} \beta_t \varepsilon_{\pi,t}^2 + \sum_{t=1}^{T} \gamma \varepsilon_{U,t}^2 \end{array} \right\}$$
(10)

Another example is a multivariate Kalman filter, which is an extension of the univariate case by taking into account additional equations, for example Phillips curve and/or Okun's law.

Kuttner's unobserved components model (Watson-Clark-Kuttner model)

The Watson-Clark unobserved component model makes assumption about the stochastic properties of the observed series and decomposes it into a trend and a cyclical component. If no other information is taken into account, the approach reduces to a purely statistical filter and interpreting the two components as potential (permanent) unemployment and transitory unemployment entirely speculative. Its advantage, however, consists in the fact that the filter allows for incorporating the structural information. Thus, the Watson-Clark-Kuttner model (also called a Kuttner unobserved component model or bivariate stochastic model) is an approach used to estimate NAIRU by combining statistical modelling (Unobserved component method based on univariate Kalman filter) and some elements of economic structure (i.e. a Phillips curve equation), thus linking cyclical component of observed time series to inflation. Kuttner complemented the Watson-Clark unobserved component model (equation (3)-(6)) with an equation that relates cycle and changes in inflation rate:

$$\Delta \pi_t = \mu_\pi + \gamma \Delta y_{t-1} + \beta_1 c_{t-1} + \theta(L) \alpha_{\pi t} \tag{11}$$

where $\Delta = 1 - L$, μ_{π} is constant, $\theta(L)$ is a moving average of order q (i.e. $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$) and $\alpha_{\pi t}$ is a Gaussian white noise variance V_{π} .

The system of equations (3)-(6) complemented by (11) is a bivariate model which can be represented in a state-space format and estimated using a Kalman filter and maximum likelihood.⁴

The model application was firstly proposed by Kuttner for the US economy; it was later used to estimate the output gap for the G7 countries and for the EMU area.

An illustration of using alternative Kalman filter and MVHP for estimating NAIRU for the 21 OECD countries is provided in (Turner *et al.*, 2001). Trends

⁴see (Durbin & Koopman, 2001), pp.115-118 for the algorithm description.

in two sets of estimates are found to be broadly similar, while MVHP estimates tend to be more closely centered around actual unemployment and Kalman filter estimates tend to be lower and suggest larger degrees of excess supply (especially for the European countries in the early to mid-1980s and mid- to late 1990s). The latter made authors conclude the Kalman filter estimates to be the preferred as being economically more appropriate.

Apel-Jansson model

While the production function approach makes use of inflation information to estimate the level of NAIRU and then exogenously inserts NAIRU into a (Cobb-Douglas) production function equation together with other input components, it does not account for the mutual dependence between unemployment gap and output gap. Apel and Jansen (Apel & Jansson, 1999) proposed an alternative approach for estimating potential output and the NAIRU, which extends Kuttner's model by additionally taking into account the mutual dependence between cyclical unemployment and cyclical output.

Identification is achieved by using a Phillips curve and Okun's law relation.

A "Triangle" Phillips model was specified by Gordon (Gordon, 1997). It explains inflation by three factors: expectations/inertia, the pressure of demand as proxied by unemployment and supply shocks.

$$\Delta \pi_t = \sum_{i=1}^3 p_i \Delta \pi_{i-1} + \sum_{j=0}^1 \eta_j (u_{t-j} - u_{t-j}^n) + \sum_{k=0}^4 w_k z_{t-k} + \varepsilon^{PC}$$
(12)

with π_t log difference of CPI, u_t the unemployment rate, u_t^n the NAIRU, z_t supply shock proxies. According to the model specification, the estimated position of the NAIRU depends on the development of actual inflation.

Okun's law relation associates cyclical unemployment fluctuations with cyclical output movements:

$$y_t - y_t^p = \sum_{l=0}^{1} \phi_l (u_{t-l} - u_{t-l}^n) + \varepsilon_t^{OL}$$
(13)

The NAIRU is assumed to follow a random walk and potential output is assumed to fallow a random walk with a drift:

$$u_t^n = u_{t-1}^n + \varepsilon^N \tag{14}$$

$$y_t^p = \alpha + y_{t-1}^p + \varepsilon^P \tag{15}$$

Though the random walk is a standard assumption in such types of models, other processes are also feasible within this framework and one can incorporate possible structural determinants of potential output and the NAIRU.

Evolution of cyclical unemployment is assumed to be a purely autoregressive process:

$$u_t - u_t^p = \sum_{m=1}^2 \delta_m (u_{t-m} - u_{t-m}^n) + \varepsilon_t^C$$
(16)

It is convenient to rewrite this model in the state space form; where unknown parameter and unobserved components are estimated by applying the Kalman filter and maximum likelihood⁵.

Combined methods are widely used for estimation of the NAIRU and output gap since the seminal contribution of Kuttner (Kuttner, 1994) who estimated a bivariate stochastic model using the US data. Apel and Jansson (Apel & Jansson, 1999) applied this methodology to Sweden, the UK, the US and Canada. In 2000 the OECD (OECD, 2000) has taken up this approach for estimation of NAIRU's for different countries. The use of the Kalman filter to estimate the NAIRU became quite popular and was applied in a number of recent studies including Gordon (Gordon, 1997), Staiger et al. (Staigner *et al.*, 1997) where it is applied to the United States, Bank of England (of England, 2000) to the United Kingdom, Gruen et al. (Gruen *et al.*, 1999) to Australia, Irac (Irac, 1999) to France, Apel and Jansson (Apel & Jansson, 1999) to Sweden and Fabiani and Mestre (Fabiani & Mestre, 2000) to the euro area.

3 Estimation procedure

3.1 Empirical framework

A combined approach built on mingling of the production function approach with the statistical filtering is used in the estimation work reported here. Based on the review of alternative theoretical approaches (a reduced variant of which was presented in Section II), a combined production function model is considered to be the most appropriate framework for estimating NAIRU and potential labor output. It is suggested to base the model on the Phillips-type relationship and estimate potential employment using two structural equations: a wage setting equation in the form of the expectation augmented Phillips curve and an employment equation allowing for supply shocks as proxied by labor force participation rate.

Following the approach used by the EC to estimate potential output of the member states in (Denis *et al.*, 2002) and (Denis *et al.*, 2006) the sustainable rate of labor utilization (N_t^*) was defined using the following structural equation:

$$N_t^* = HRS_t^* \times PWA_t \times PR_t^{HP*}(1 - U_t^*) \tag{17}$$

where HRS_t^* is an index of trend hours worked, PWA_t is the population of working age, PR_t^{HP*} is the trend participation rate and U_t^* is the trend unemployment rate (NAIRU). All the star-variables are assumed to be endogenous.

Labor input, according to the methodology, is decomposed into the number of employees and the average hours worked per employee which provides a more meaningful measure for the rate of technological progress. In many previous estimation exercises TFP was biased downwards due to the secular decline in the average hours worked per employee. Introduction of hours worked affects

 $^{{}^{5}}$ See, for example, (OECD, 2000) for the model application.

how the potential growth is attributed to the various factors of production, especially labor and TFP, with TFP in general being increased and labor being correspondingly reduced.

The estimation procedure of potential employment is reduced to determining the trend of labor input and is performed in several steps. The definition of trend labor input is started from the maximum possible level, namely the population of working age (PWA_t) . From here the trend labor force was calculated by mechanically detrending (using HP filter) the participation rate (PR_t) .

$$LF_t = PR_t^{HP} \times PWA_t \tag{18}$$

In a next step the trend unemployment, consistent with the NAIRU, is obtained as follows. The NAIRU is estimated from the modified Phillips curve equation ("triangle" Phillips model) using the Kalman filter. A specialized program GAP with a convenient interface for specifying and estimating the Watson-Clark-Kuttner model was developed by C.Planas and A.Ross (Planas & Rossi, 2004). This program was used in the current estimation exercise to compute the unobserved NAIRU time series for different regions in Belgium and at the national level.

The idea behind Kalman filtering when applied to the estimation of the NAIRU is essentially the following. The observed unemployment rate (U_t) is decomposed into a trend (U_t^T) and cyclical (U_t^C) components:

$$U_t = U_t^T + U_t^C \tag{19}$$

Both components are treated differently

• U_t^T (NAIRU) is estimated by a time-series model, which captures its general statistical properties (like, for example, non stationarity of structural unemployment). Economic information which can explain structural unemployment is regarded as unobservable.

• U_t^C (unemployment gap, i.e. observed unemployment minus NAIRU) is modelled with the use of economic information: the link between changes in inflation and cyclical unemployment is derived from the Phillips curve.

The trend component of unemployment is assumed to have a stochastic trend 6 and is modelled as a random walk with drift :

$$U_t^T = \mu_t + U_{t-1}^T + z_t \tag{20}$$

The drift itself is allowed to be stochastic and follows a random walk:

$$\mu_t = \mu_{t-1} + \alpha_t \tag{21}$$

In order to estimate the *cyclical* component of unemployment the Phillips curve equation is rewritten adding exogenous variables:

⁶It is more appropriate to model economic time series as having stochastic rather then deterministic trend because high predictability implied by deterministic trend is hardy associated with economic forces that can cause shifts in unemployment. Large unpredictability assumes a random component in the model.

$$\Delta \pi_t = \alpha + \gamma X_t + \beta U_t^C + e_t \tag{22}$$

where $\Delta \pi_t$ is a change in wage inflation, $U_t^C = (u_t - nairu_t)$ is unemployment gap, X_t is a vector which contains exogenous variables such as wage inflation, changes in the participation rate and work mobility between regions; other unobserved shocks are captured by the error term e_t .

After having estimated the NAIRU, it is possible to calculate potential employment and correct obtained results for the trend in hours worked.

3.2 Data

Data used for this project comes from different sources. While some time series in hand go back to 1980, data on other variables was put available only for the later period. As a result, the estimation period when all available series have common time characteristics, covers 23 years starting from 1985 up to 2007. The observed harmonized rate of unemployment at the national level (for the whole estimation interval) and at the regional level (for the recent years, 1999-2007) was obtained from the National socioeconomic database of the National Bank of Belgium (Belgostat); older regional series come from the National Institute of Statistics.

Data on population of working age, employment and unemployment (used to calculate the participation rate) comes from the Belgian Labour Force Survey and was obtained from the National Institute of Statistics. Labour force participation rate plays a central role in the study of the size and dynamics of country's human resources. Derived information can also be used to formulate employment policies and rates of accession to (and retirement from) economic activity, which is crucial for national and regional financial planning of social security systems⁷.

Time series of the labor mobility between regions were provided by the National Institute of Statistics. It was decided to incorporate this indicator as an exogenous variable into the structural equation used to estimate the NAIRU since work commuters flows is an unambiguous characteristic of the Belgian labor market.

Changes in inflation (in the expectations-augmenting modified Phillips curve) were measured by the wage inflation instead of CPI. Besides simple reasoning built on the higher significance of the wage inflation index (comparing to the CPI) considering the research objectives, the decision to use this economic indicator for estimating the NAIRU was also motivated by the fact that, under the assumption of rationality of labor market participants and considering the official yearly indexation of labor income, wage setting fully incorporate real inflation in Belgium. Since data on wage inflation at the regional level is not available, national time series were used for all three regional models. It can be argued that imposing wage inflation equality between different regions might

 $^{^7{\}rm Key}$ Indicators of the Labour Market (KILM) 5th Edition software http://www.ilo.org/public/english/employment/stratkilm/download/kilm01.pdf

have caused certain bias in the estimation results. However, such inexactness of the regional wage inflation estimates can be considered insignificant since official wage settings (such as, for example, minimum wage and unemployment benefits rate) are fixed within the country while the main divergence between regional labor markets consist in the different employment opportunities and the job structure.

3.3 Estimation results

The NAIRU estimation is performed by means of the GAP program (Planas & Rossi, 2004), which allows to compute an unobserved component of historical time series and to forecast the future behavior of the NAIRU depending on the exogenously imposed shocks. Cyclical component of unemployment is estimated via the Phillips curve modified equation (see Annex B for the detailed explanation) which incorporates changes in participation rate and work mobility (commuting) between regions as exogenous variables.

While there are not many doubts about casual link between the participation rate and the NAIRU dynamics, inclusion of commuting into the equation demands an additional explanation. High work mobility is a specific feature of the Belgian labor market. Especially large flows of commuters are observed between Brussels and Flanders. Figures in Table 1 witness a substantial number of interregional work commuters. Thus, the biggest home-work flows in 2007^8 were observed in the following directions: Brussels-Flanders (10.46%), Flanders-Brussels (8.74%) and Wallonia-Brussels (9.44%). Inclusion of work mobility as an exogenous variable into the second equation helps to achieve a better fit of the model to the regional labor markets behavior. Besides, estimation of the regional mobility rate coefficient (statistically significant) provides a valuable policy implication.

Work mobility was incorporated into the Phillips curve equation by means of two different coefficients alternatively: the mobility rate and the share of "in" commuters in the total number of active jobs in the region. Mobility rate coefficients were provided by the NIS and show the share of mobile workers, who work outside their region of residence, in the total number of employed in regions and in the country as a whole. In order to provide insight in the principal directions and nature of the labor mobility between different Belgian regions we constructed another measurement of the commuting rate - a share of "in" commuters in the total number of active jobs in the region. Dynamics of both coefficients is presented in Figure 1 in Appendix C. It can be clearly seen from the Figure 1b, that main inflows of commuters take place in Brussels.

While the classical supply-oriented explanation of work migration (workers move towards more prosperous regions) should not be neglected, suburbanization, however is considered to be more important in explaining impressively high inflows of commuters towards Brussels. More explicitly, a rising welfare has created the possibility for more workers to buy family dwellings and move out from

⁸ as a share of the total working population of the region

Brussels. Accompanied by the growing efficiency and affordability of transport infrastructure, this process considerably reshaped the population composition of Brussels and surrounding Flemish areas: many high-skilled workers have chosen to reside outside Brussels and spend more time on travelling to their work places, while low-skilled workers and low-income ethnic minorities stayed in the capital. This can explain the high level of unemployment rate observed in Brussels, accompanied by the highest level of commuters inflows among all Belgian regions.

The estimation output of the model for different regions and for Belgium as a whole is presented in Table 2 and Figure 2. As it can be clearly seen from the graphs, the NAIRU dynamics differs across the regions. The effect of high NAIRU in Wallonia and high and growing NAIRU in Brussels is partly offset by the comparatively low and decreasing NAIRU in Flanders, which as a result provide stability of NAIRU time series at the national level. Model provides a good fit to the national and regional data on actual unemployment (R-squared is above 90 per cent in all the cases) and our results for a country as a whole look pretty similar to the ones obtained by the EC (ECFIN, 2006).

Two estimation sets have been obtained using alternative measurement of the working migration, as explained above. The NAIRU in both estimation sets do not show essential dissimilarities. The only disparity is that when work mobility is measured by the share of in-commuter in the total number of active jobs, the NAIRU estimates tend to be more closely centered around actual unemployment, which is quite intuitive since inflows of commuters are more closely related to the unemployment rate in the region then out-commuters' migration dynamics. Estimation results are presented in Table 3 (with *nairu1* - estimates obtained using mobility rate and *nairu2* - using the share of "in" commuters of the total number of active jobs in the region).

Since the most pronounced reason for the labor force changes is demographics, estimation sample was further extended in order to check for the age-specific changes in the NAIRU. The breakdown of the Belgian labor force by age group for each region gives a profile of the distribution of the economically active population within a country. Three cohorts including male and female workers aged 15-24 (teenagers and young adults), 25-49 (prime age adults) and 50-64 years (older adults) were used to estimate cohort-specific NAIRUs for each region and for the whole country. As a result, quite different estimates of the regional NAIRUs within each cohort were obtained, with the highest NAIRU being observed in Brussels and the lowest in Flanders (Figure 3). These results go in line with two trends in the Belgian population change⁹: growth of population in the Brussels and Flemish functional urban regions and decline or stagnation in the Walloon ones.

Thus, if we look at the youngest cohort, it can be noticed that the national NAIRU is slightly increasing, showing dynamics which are different from the 15-64 years sample. Recall from the previous estimation results that the national NAIRU shows an upward sloping trend with a high NAIRU in Wallonia, high

⁹documented, for example, in (Aujean *et al.*, 2005).

and increasing in Brussels and comparatively low and decreasing in Flanders. Now, instead, the model produces an almost constant NAIRU in Flanders and an increasing NAIRU in other two regions, with Brussels estimates showing three major tweaks - in 1991, 1999 and 2002. Growth of the NAIRU in Brussels in 1990s can be explained by the changing age structure as the baby boom generation has moved through the labor force and by second-wave of migration, when younger cohorts followed up their parents once the later have settled and made initial living and work arrangements. For the prime-age adults the model appears to have captured the general trends and even the point estimates show persistent similarity. For age cohorts between 50 and 64 years the NAIRU for Brussels and Walloon region show steady resemblance with a growing trend from 1991 till late 90s followed by a decrease till 2000, and a persistent growth till the present time.

Obtained NAIRU estimates were used to calculate potential employment according to equation (17). After applying HP filtering to the detrend participation rate and average hours worked time series, the potential employment consistent with the NAIRU was computed. Determinants of labor potential are presented in Table 4, while the dynamics of potential employment for Belgium and for three regions is shown in Figures 4 and 5. As whitnessed by figure 5, working hours supplied to the national and region labor markets by the prime age adults did not change on average. Potential employment of the population of working age did not show visible changes in Brussels and was roughly stable with a light dip in 1999-2000 and an upward trend thereafter for other regions. Teenagers and young adults in Flanders were supplying nearly 1.5 times less labor in 2000s compared to 1986, which can be attributed to the growing rates of the full time enrolment into the higher education and training for these cohorts. Potential employment of the older adults started growing in the late 1990s in Flanders and to the less extent in Walloon region.

4 Conclusions

This paper has provided different estimates of the NAIRU for Belgian regions using a so called production function approach - a combination of structural equations with statistical modelling. Obtained results led to several main conclusions.

First, this research contributes to the wide range of literature which proves the feasibility of fusing statistical filtering and economical structure in the estimation of unobserved variables and links empirical technique used to obtain NAIRU estimates to the theoretical concept of the unemployment consistent with the stable inflation.

Second, inclusion of demographic and labor mobility component into the Phillips curve modified equation allows to account for the country specific features of the Belgian labor markets and to obtain feasible estimates of the regionand age-specific NAIRUS. Among others, obtained results confirm a hypothesis of inverse relationship between regional unemployment and willingness to commute. Assuming that the main reason for commuting is search for better employment opportunities and higher wages, it can be concluded that higher returns to work play a role of compensating differential for unemployment. An estimation exercise reported in this paper finds a significant divergence in both levels and trends of the NAIRU between three Belgian regions. It follows that reduction of regional unemployment disparities will lead to the higher economic efficiency as measured by the regional and national output. Region- and age group-specific estimates offer important policy insight in unemployment and can help in choosing the right labor market policy instruments for specific target groups of population.

As with any research project, we found that there was a lot more information to take in than could be covered in a project of this size, providing a promising potential for further research. One of the further research opportunities can be based on inclusion of region-specific variables which capture interregional variation in the wage setting: density of and coverage by unions and industrial structure of regional economies. Such an extension can probably involve a broader specification of structural elements of the model and can prove powerful in explaining long term unemployment disparities between Belgian regions by location of declining or (potentially) growing industries in certain functional urban regions.

References

- APEL, M, & JANSSON, P. 1999. System estimates of potential output and the nairu. *Empirical economics*, 24(3), 373–38.
- AUJEAN, L., CASTIAU, E., ROELANDTS, M., & VANDERMOTTEN, C. 2005. Analysis of policy documents & policy focus groups: Central belgium. Polynet action 3.1. Institute of Community Studies/The Young Foundation and Polynet Partners., London.
- BLANCHARD, O.J., & QUAH, D. 1989. The dynamic effects of aggregate demand and supply disturbances. *American economic review 79*.
- CHIARINIA, B., & PISELLI, P. 2000. Identification and dimension of the nairu. Economic modelling.
- CLARK, P.K. 1987. The cyclical component of u.s. economic activity. *Quarterly* journal of economics, 102(4), 797Ű814.
- COTIS, J., ELMESKOV, J., & MOUROUGANE, A. 2005. Estimates of potential output: Benefits and pitfalls from a policy perspective. Tech. rept. OECD Economics Department.
- DENIS, C, MC MORROW, K, & ROGER, W. 2002. Production function approach to calculate potential growth and output gaps estimates for the eu member states and the us. Economic Papers 176. EC.
- DENIS, C, GRENOUILLEAU, MC MORROW, K, & ROGER, W. 2006. Calculating potential growth rates and output gaps - a revised production function approach. Economic papers. EC.
- DURBIN, J., & KOOPMAN, S. J. 2001. *Time series analysis by state space methods*. Oxford University Press.
- ECFIN. 2006. Reference manual for the ecfin's production function methodology. Tech. rept. EC.
- FABIANI, S., & MESTRE, R. 2000. Alternative measures of the nairu in the euro area: estimates and assessment. Working Papers 17. European Central Bank.
- FUNKE, M. 1997. Supply potential and output gaps in west german manufacturing. *International journal of forecasting*, **13**.
- GORDON, R.J. 1997. The time-varying nairu and its implications for economic policy. *Journal of economic perspectives*, 1, 11–32.
- GRUEN, D., PAGAN, A., & THOMPSON, C. 1999. The phillips curve in australia. *Journal of monetary economics*, 44(2).

- HJELM, GÖRAN. 2003. Simultaneous determination nairu, output gaps, structural budget balances: Swedish evidence. Working Paper 81. The National Institute of Economic Research, Stockholm.
- IRAC, D. 1999. *Estimation of a time varying nairu for france*. Ecb workshop on structural unemployment. Banque de France.
- KUTTNER, K.N. 1994. Estimating potential output as a latent variable. *Journal* of business and economic statistics, **12** (3).
- LAXTON, D., & TETLOW, R. 1992. A simple multivariate filter for the measurement of potential output. Technical Report 59. Bank of Canada.
- OECD. 2000. Revised oecd measures of structural unemployment. Economic outlook. OECD.
- OF ENGLAND, BANK. 2000. *Economic models at the bank of england*. London: Bank of England.
- PLANAS, C., & ROSSI, A. 2004. Program gap. Technical description and usermanual. EC, Joint Research Center.
- RICHARDSON, P., BOONE, L., GIORNO, C., MEACCI, M., RAE, D., & TURNER, D. 2000. The concept, policy use and measurement of structural unemployment: Estimating a time varying nairu across 21 oecd countries. Technical report 250. OECD, Paris.
- STAIGNER, D., STOCK, J.H., & WATSON, M.W. 1997. How precise are estimates of the natural rate of unemployment?, in reducing inflation: motivation and strategy. Chicago: University of Chicago Press.
- TURNER, D., BOONE, L., GIORN, C., MEACCI, M., RAE, D, & RICHARD-SON, P. 2001. Estimating the structural rate of unemployment for the oecd countries. Economic studies. OECD.
- WATSON, M.W. 1986. Univariate detrending methods with stochastic trends. Journal of monetary economics, 18, 49–75.

A General specification of the state space model

State space model is a way to subsume a whole class of special cases of interest in time series (in much the same way that linear regression does). Was firstly introduced in Kalman (1960) and Kalman and Bucy (1961). The model was originally introduced for the use in aerospace-related research, but later was largely applied to modeling data in economics.

The model employs a first order autoregressive vector as the state equation, which determines the rule for the generation of the unobserved state vector x_t with dimension $p \times 1$ from the past state vector x_t with the same dimension:

$$x_t = \Phi x_{t-1} + w_t \tag{23}$$

It is assumed that w_t is $p \times 1$ dimensioned, independent and identically distributed with zero mean and covariance matrix Q.

Since the state vector x_t can not be observed directly but through its linear transformation with noise added, state space model adds an observation equation of the form

$$y_t = A_t x_t + v_t \tag{24}$$

where A is a $q \times p$ observation matrix. Output vector (or vector of observations) y_t has a dimension $q \times 1$ and contains an observed data. Observed series can be larger or smaller then p, the dimension of the underlying series of interest. Noise v_t is assumed to be white and Gaussian with $q \times q$ covariance matrix R. w_t and v_t are assumed to be uncorrelated.

An underlying idea is that the development of the system over time is determined by x_t according to 23 The basic model can be extended by including exogenous variables into the state or into the observation equations. For example, if we assume to have a $r \times 1$ vector of inputs u_t , then the state space model takes the form of:

$$x_t = \Phi x_{t-1} + \Psi u_t + w_t$$

$$y_t = A_t x_t + \Gamma u_t + v_t$$
(25)

with $p \times r$ dimension Ψ and $q \times r$ dimension Γ .

An advantage of this approach is related to the possibility to

· estimate the unknown parameters contained in A_t , Φ , Ψ , Q and R, that define the particular model, and

 \cdot estimate or forecast values of the underlying unobserved process x_t .

Missing data configurations can be treated and a vast spectre of additional models can be generated. Another advantage lies in the analogy between the observation matrix A_t and the design matrix in usual regression allows to generate fixed and random effect structures (either constant or time-varying) simply by making appropriate choice for the matrix A_t and the transition structure Φ .

Forecasting, Filtering and Smoothing

In practice the state space models as defined by 23 and 24 or 25 are used to estimate the underlying unobserved variable x_t for time s given the data $Y_s = \{y_1, ..., y_s\}.$ When s < t, the problem is called *forecasting (or prediction)*.

When s = t, the problem is called *filtering*.

When s > t, the problem is called *smoothing*.¹⁰

In brief, Kalman filter generates, for a given set of model parameters and starting values, a sequence of optimal conditional predictions of the observable variables. The prediction errors are then used in a maximum likelihood routine to find the optimal set of parameters and the corresponding estimates of unobserved parameters.

¹⁰All abovementioned probelms can be solved using Kalman Filter and Kalman Smoother.

B General specification of the model used for the NAIRU estimation.

1st equation

Is specified similarly to regression models with ARIMA errors ¹¹

$$U_{1t} = \sum_{i=1}^{M_1} \alpha_{1i} Z_{1it} + \widetilde{U_{1t}}$$
(26)

where Z_{1it} is a vector of $M_1 \leq 10$ exogenous variables. The reminder of this regression, U_{1t} is described as made up of a long term component (trend), U_{1t}^T , and of a short term (cyclical) component U_{1t}^C according to:

$$\widetilde{U_{1t}} = U_{1t}^T + U_{1t}^C \tag{27}$$

Trend component (NAIRU) is assumed to have a stochastic trend and is modelled as a random walk with drift, drift itself is a stochastic trend and follows a random walk:

$$U_{1t}^T = \mu_t + U_{1t-1}^T + v_t^T \tag{28}$$

$$\mu_t = \mu_{t-1} + \alpha_t \tag{29}$$

Cyclical component is AR(2):

$$(1 - \phi_1 L - \phi_2 L^2) U_{1t}^C = v_t^C \tag{30}$$

 v_t^T and v_t^C are permanent and transitory shocks, independent Gaussian white noises (white shock innovations) with variances V_T and V_C respectively.

Exogenous variables are assigned to the trend component U_{1t}^T so that the final decomposition is:

$$U_{1t} = U_{1t}^{TF} + U_{1t}^C \tag{31}$$

where the final trend U_{1t}^{TF} is such that:

$$U_{1t}^{TF} = \sum_{i=1}^{M_1} \alpha_{1i} Z_{1it} + U_{1t}^T$$
(32)

Putting it all together:

$$U_{1t} = \sum_{i=1}^{M_1} \alpha_{1i} Z_{1it} + U_{1t}^T + U_{1t}^C$$
(33)

2nd equation

The model is complemented (following Kuttner) with an additional transition equation that incorporates an economic structure and relates the cyclical

¹¹Planas Ch. and Rossi A. Program GAP, Version 3.1. Technical Description and User-Manula, February 2004, p.4.

component (change in unemployment) U_{1t}^C and inflation. In other words, series 1 is related to stationary transformation of the second series, *i.e.* change in inflation:

$$\Delta \pi_{2t} = \mu_{2\pi} + \sum_{i=1}^{p} \alpha_{2i} Z_{2it} + \beta_i U_{1t-i}^C + \sum_{i=1}^{p} \phi_i \pi_{2t-i} + \sum_{i=0}^{q} \theta_i v_{t-i}^{\pi}$$
(34)

where $\Delta = 1-L$,

 $\mu_{2\pi}$ is the intercept,

 Z_{2it} is a vector of exogenous variables

p is the number of autoregressive terms constrained to be less or equal to 2; in our case p = 2.

q is the number of moving average terms with 3 as maximum and $\theta_o = 1$

 v_t^{π} is a white noise innovation with variance V_{π} that can be correlated with v_t^S but only when $\beta o = 0$

	place of work:									
	Brussels		Flanders		Wallonia		Abroad		Total	
	1000s	%	1000s	%	1000s	%	1000s	%	1000s	
residence:										
Brussels	321	84.00	40	10.46	16	4.29	5	1.25	383	
Flanders	236	8.74	2386	88.47	24	0.90	51	1.89	2697	
Wallonia	123	9.44	40	3.08	1089	83.71	49	3.78	1300	
Total	680	15.52	2466	56.31	1129	25.78	105	2.39	4380	

C Tables and Figures

Table 1: Work mobility in Belgium, 2007

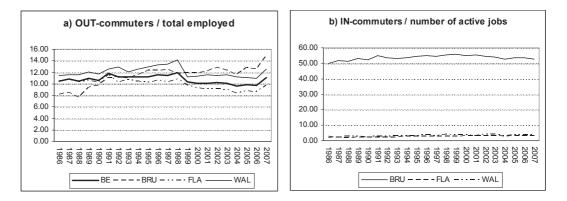


Figure 1: Labor mobility

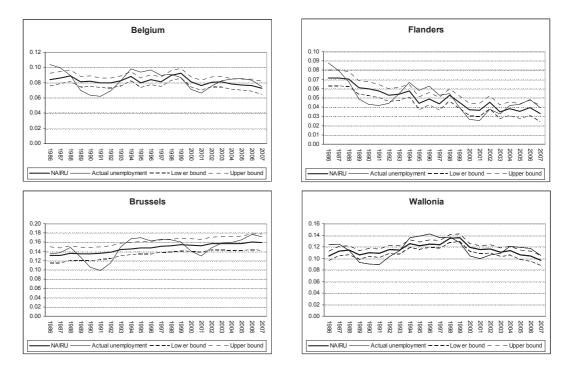
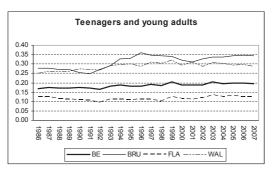
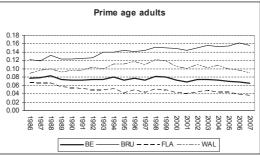


Figure 2: NAIRU estimates and observed unemployment





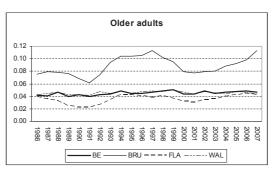


Figure 3: NAIRU estimates within age cohorts

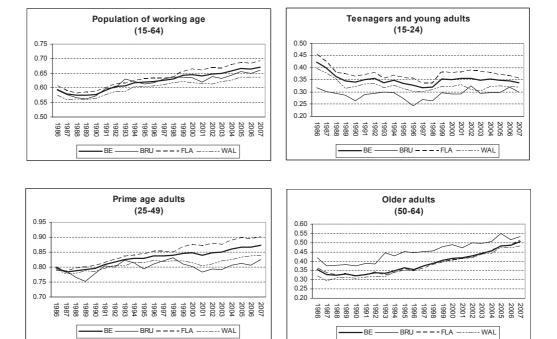


Figure 4: Labor participation rate

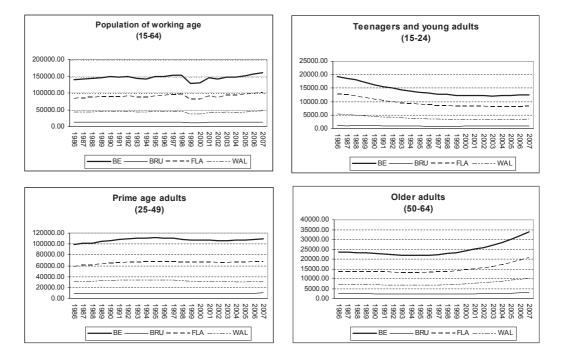


Figure 5: Potential employment

	Belgiun	1								
	coefficient	standard error	t-statistic							
lagged wage inflation	0.1796	0.1852	0.9698							
participation rate	-0.0146	0.0436	-0.3359							
mobility rate	-0.5455	0.3362	-1.6222							
R-squared	0.9289									
No of observations	22									
Period	1986-2007									
	Brussel	5								
	coefficient	standard error	t-statistic							
laged wage inflation	0.3639	0.2221	1.6390							
participation rate	-0.1697	0.0989	-1.7150							
mobility rate	0.1685	0.1313	1.2833							
R-squared	0.9297									
Flanders										
laged wage inflation	0.2521	0.1797	1.4027							
participation rate	-0.0525	0.0433	-1.2128							
mobility rate	-0.2466	0.3223	-0.7650							
R-squared	0.9277									
Wallonia										
laged wage inflation	0.2000	0.1691	1.1832							
participation rate	0.0821	0.0761	1.0780							
mobility rate	-0.5676	0.2434	-2.3318							
R-squared	0.9360									
No of observations	22									
Period	1986-2007									

Table 2: Determinants of labour potential

		BRU			FLA			WAL	
	unempl	nairu1	nairu2	unempl	nairu1	nairu2	unempl	nairu1	nairu2
1986	13.47	13.35	13.25	8.77	7.15	7.10	12.41	10.36	10.44
1987	13.73	13.31	13.19	7.98	7.15	7.11	12.46	11.16	11.21
1988	14.88	13.73	13.65	6.76	7.01	6.98	11.37	11.31	11.33
1989	12.80	13.60	13.50	4.90	6.12	6.10	9.29	10.49	10.50
1990	10.54	13.62	13.53	4.29	5.99	5.97	9.04	10.99	10.99
1991	9.86	13.73	13.66	4.19	5.74	5.71	8.94	10.91	10.92
1992	11.46	13.91	13.86	4.42	5.30	5.27	10.35	11.60	11.63
1993	15.01	14.38	14.38	5.49	5.43	5.40	11.39	11.52	11.55
1994	16.70	14.52	14.54	6.68	5.76	5.74	13.62	12.61	12.63
1995	16.94	14.70	14.73	5.80	4.43	4.42	13.89	12.19	12.18
1996	16.23	14.73	14.75	6.26	4.89	4.93	14.24	12.46	12.41
1997	16.67	15.03	15.08	5.27	4.37	4.45	13.56	12.33	12.25
1998	16.49	15.13	15.19	5.46	5.27	5.40	13.68	13.37	13.27
1999	16.06	15.37	15.43	3.99	4.56	4.70	12.78	13.47	13.38
2000	14.07	15.27	15.31	2.68	3.73	3.83	10.42	11.84	11.79
2001	12.98	15.19	15.21	2.58	3.68	3.71	9.95	11.49	11.48
2002	14.73	15.56	15.62	3.77	4.52	4.47	10.61	11.68	11.70
2003	15.76	15.65	15.71	3.27	3.49	3.34	10.87	11.11	11.15
2004	15.90	15.61	15.66	4.19	3.83	3.64	12.06	11.41	11.46
2005	16.47	15.66	15.72	4.33	3.56	3.39	11.93	10.71	10.76
2006	17.69	15.91	15.99	4.82	3.93	3.85	11.75	10.45	10.50
2007	17.17	15.84	15.91	3.96	3.30	3.33	10.53	9.67	9.69

Table 3: Determinants of labour potential

year	growth of the PWA					trend PR $(\%)$				NAIRU			
	BE	BRU	FLA	WAL	BE	BRU	FLA	WAL	BE	BRU	FLA	WAL	
1986	х	х	х	х	58	57	59	56	8.43	13.35	7.15	10.36	
1987	1.3	0.2	1.7	-0.2	58	58	59	56	8.68	13.31	7.15	11.16	
1988	0.0	0.3	0.1	0.6	58	58	59	57	8.91	13.73	7.01	11.31	
1989	0.3	-0.3	0.3	0.0	59	58	60	57	9.16	13.60	6.12	10.49	
1990	-0.1	-0.8	0.0	-0.1	59	59	60	58	8.19	13.62	5.99	10.99	
1991	-0.1	-1.4	0.1	0.3	60	60	60	58	7.96	13.73	5.74	10.91	
1992	0.2	-0.4	0.2	0.2	60	60	61	59	7.95	13.91	5.30	11.60	
1993	0.2	-0.1	0.3	0.1	61	61	61	59	8.22	14.38	5.43	11.52	
1994	0.1	-0.1	0.2	0.0	61	61	62	60	8.84	14.52	5.76	12.61	
1995	0.1	-0.3	0.2	-0.2	62	62	63	60	7.99	14.70	4.43	12.19	
1996	0.0	0.3	0.0	0.0	62	62	63	60	8.38	14.73	4.89	12.46	
1997	0.1	0.4	0.1	0.0	63	62	64	61	8.15	15.03	4.37	12.33	
1998	0.0	0.4	0.0	0.0	63	63	64	61	8.96	15.13	5.27	13.37	
1999	0.1	0.2	0.0	0.1	64	63	65	61	9.24	15.37	4.56	13.47	
2000	0.1	0.7	0.0	0.2	64	63	66	62	8.12	15.27	3.73	11.84	
2001	0.1	0.6	0.0	0.2	65	64	66	62	7.62	15.19	3.68	11.49	
2002	0.4	1.7	0.2	0.5	65	64	67	62	8.06	15.56	4.52	11.68	
2003	0.5	1.9	0.3	0.5	65	64	67	63	8.09	15.65	3.49	11.11	
2004	0.4	1.0	0.2	0.6	66	65	68	63	7.82	15.61	3.83	11.41	
2005	0.8	1.5	0.6	1.1	66	65	68	63	7.68	15.66	3.56	10.71	
2006	0.9	1.6	0.8	1.0	67	65	69	63	7.61	15.91	3.93	10.45	
2007	1.0	1.5	0.8	1.0	67	66	69	64	7.27	15.84	3.30	9.67	

Table 4: Determinants of labour potential







