Spoor A2:

Estimating the NAIRU and potential employment for Belgian regions.

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1 Introduction

The value of the non-accelerating inflation rate of unemployment (NAIRU) for policy questions is keeping on gaining its importance in the last two decades. Setting an economic policy requires, among others, an identification of the sustainable rate of capacity utilization. In this sense susceptibility is normally associated with reasonably stable inflation. Thus, in terms of labour market the idea of sustainable resources utilization is closely connected to the concept of potential employment and the NAIRU which provides a benchmark to identify sustainable and unsustainable trends in unemployment and output. Short term employment-stimulating economic policies alone can not permanently effect equilibrium unemployment: the NAIRU may, to some degree, be influenced by the path of actual employment, but its shifts are mainly determined by structural factors. Temporary supply shocks may alter the rate of inflation, while the NAIRU will be basically unaffected once they are outdone; by contrast, long-term supply shocks (caused, for example, by demographic changes) may permanently modify the NAIRU path.

Despite the prominence of the NAIRU concept, its empirical implementation often provides controversial results since the variable itself is unobservable and not always well defined. As a result, alternative methodologies for measuring the NAIRU can lead to important divergences in their estimates. Country administrations and international economic authorities, however, find it useful to use NAIRU estimates as an input to various assessments of fiscal and structural policies. Model specification and choice of estimation method become an important issue in given context.

In this paper we present a review of alternative theoretical approaches broadly classified into three groups: statistical, structural and combined methods. Conceptual basis of different assessments and their ability to provide precise estimates are analyzed with respect to the usefulness for economic policy analysis.

Since the conventional wisdom and officially adopted approach of the leading international economic organizations for estimating the NAIRU relies on the existence of an expectations-augmented Phillips-type relationship, we suggest to model potential labour output using two structural equations: wage setting equation in the form of expectation augmented Phillips curve and an employment equation allowing for supply shocks as proxied by labour force participation rate. The NAIRU is estimated from the modified Phillips curve equation ("triangle" Phillips model) using the Kalman filter.

2 Review of Existing Techniques

Being an unobserved variable, NAIRU depend on a wide range of economic and institutional factors and it follows that it can only be estimated with uncertainty. A wide range of methods have been developed for this purpose. Here we suggest a short review of the most commonly used techniques and discuss their advantages and pitfalls. While it should be admitted that methods classification is rather questionable, three groups are usually made out from the variety of approaches[1]:

- statistical (or time-series)
- economical (or structural)
- combined (a compromise between the previous two)

Although the approaches differ, they share the common aim to remove business-cycle affects from the observed time series for unemployment.

2.1 Statistical Methods

Trending Methods.

Trending methods suggest to decompose actual unemployment into a deterministic trend component and a cyclical component, with the latter identified as the NAIRU. The trend component of unemployment is supposed to be a linear function of time. It thus involves a linear regression of the log unemployment on a constant and a time trend, with e_t – demand shocks

$$\ln U_t = \alpha + \beta t + e_t \tag{1}$$

The NAIRU in this equation is given by $(\alpha + \beta t)$, while potential unemployment growth rate is estimated by the slope (β) and is supposed to be constant.

Among the most cited pitfalls of using this estimation technique are the following three: trending can not allow for any supply shock; a constant NAIRU is implied; the cyclical unemployment can be biased by partially allocating trend components into the cyclical component (as the stochastic trend is not fully eliminated when the resulting gaps are not stationary)

Univariate filters

[1] There are still some overlaps in such classification. See Cotis, J., Elmeskov, A. and Mourougane, A. (2005) Estimates of Potential Output: Benefits and Pitfalls from a Policy Perspective. OECD, Economics Department, p.9.

The objective of filtering is to update our knowledge about the system each time a new observation is brought in (filtering = detrending).

a) *Hodrick Prescott filter*

Filtering is performed by introducing a trade off between a good fit to the actual series (1st term in equation (2)) and the degree of smoothness of the trend series (2nd term in equation (2)).

$$HP = \min \left\{ \begin{array}{c} \sum_{t=1}^{T} (\ln U_t - \ln U_t^*)^2 + \\ +\lambda \sum_{t=1}^{T} \left(\left[\ln U_{t+1}^* - \ln U_t^* \right] - \left[\ln U_t^* - \ln U_{t-1}^* \right] \right)^2 \end{array} \right\} \tag{2}$$

where U_t and U_t^* are actual unemployment and trend unemployment (or NAIRU) respectively and λ is Lagrange multiplier.

The method has been criticized mostly for imprecisely estimates of the time series end points and an absence of a common rule for choosing an objectively correct value for the parameter λ. Besides, Hodrick Prescott (HP) filter can not capture structural breaks in the trends of economic series. One of the most commonly used improvements of the HP filter is an extension of the forecast period over the data sample in order to mitigate the end-of-sample problem.

b) *Watson-Clark unobserved components model*

Unobserved component (UC) model was developed by Watson (1986) and Clark (1987) and assumes that macroeconomic time series contain trend, cycle and, in some cases, erratic components which are not directly observable. Decomposition into these three elements can be performed by imposing sufficient restrictions on the trend and the cycle. For example, the log of observed unemployment is assumed to be:

$$u_t = u_t^p + c_t \tag{3}$$

The first term stands for the permanent component and the second for the cyclical one. The two components are not correlated with each other.

The *permanent component* can be seen as an estimate of NAIRU and is often specified as

$$u_t^p = u_{t-1}^p + \mu_{t-1} + \eta_t \tag{4}$$

$$\mu_t = \mu_{t-1} + \varsigma_t \tag{5}$$

with η_t and ς_t orthogonal white noises.

The *transitory cyclical component* is unobserved stationary AR(2):

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \gamma_t \tag{6}$$

where γ_t is a white noise and ϕ is an unknown parameter which in classical analysis is replaced by its maximum likelihood estimate.[2]

[2] In terms of state space model (see later in the Section): equation (3) is an observation equation, equations (4)-(5) are transition equations.

UC model is usually estimated by the Kalman filter, which has two distinct phases: predict and update. The predict phase uses the state estimate from the previous timestep to produce an estimate of the state at the current timestep. In the update phase, measurement information at the current timestep is used to refine this prediction to arrive at a new, (hopefully) more accurate state estimate, again for the current timestep.

Univariate filters have been questioned for their ability to appropriately distinguish between the permanent and transitory components of the time series.

The basic problem with all statistical methods is that they do not involve economic prior and depend on arbitrary assumptions in order to perform decomposition into trend and cyclical components. For example, in the case of HP filter trend unemployment is identified as a weighted moving average of actual unemployment, while in a Kalman Filter it is assumed to be a random walk. Another problem is connected with the fact that the indicators obtained are not well defined since all the information other then observed unemployment is ignored and leads to the trend estimates which are usually centered around actual time series. Another serious drawback is due to the difficulty to judge the degree of precision of the result in most cases.

2.2 Structural Methods

Structural methods attempt to estimate NAIRU by means of aggregate structural models of wage and price setting behavior. This approach requires a strong assumption of full or partial market equilibrium where wages are bargained between workers and firms and latter decide on the level of employment, prices and output after a wage agreement has been reached. Product market conditions, capital stocks and technology are assumed to be exogenously given.

An example of structural model is presented bellow. This model suggests to estimate a system of equations describing wage, price settings and labour supply as follows[3].

Price equation:

$$p - w = \alpha_0 + \alpha_1 n + \alpha_2 \Delta n + \alpha_3 (p - p^e) - le + LT^p + ST^p \tag{7}$$

Wage equation:

$$w - p = \beta_0 - \beta_1 U - \beta_2 \Delta U - \beta_3 (w - w^e) + le + LT^w + ST^w \tag{8}$$

Labour supply

$$l = \gamma_0 - \gamma_1 U + LT^l \tag{9}$$

where U is unemployment, p, w, n, l and le are respectively prices, wages, employment, labour force and trend labour efficiency in the logarithmic form,

[3] Richardson et al (2000)

Δ is a first difference operator, and LT and ST are the vectors of exogenous variables which represent long-term and short-term shocks.

Another example of structural models is a structural autoregressive model (*SVAR).* This model was developed by Blanchard and Quah (1989). The method is based on a structural autoregressive model that estimates potential output and the output gap using structural assumptions about the nature of economic disturbances. The model uses the information about employment and capacity utilization to decompose actual time series into a permanent trend component (supply) and temporary cyclical component (demand). Each of the three variables is regressed in the system on their own lags and the lags of the other variables. The shocks are grouped into supply and demand shocks. The key assumptions of the model are that demand shocks do not affect output in the long run whereas supply shocks do. The variables included in the different applications differ but in every case the main objective is to remove fluctuations due to demand conditions from the output series in order to determine potential output. Potential output is calculated by historical decomposition addressing the question: what would the output series look like in the absence of demand shocks.

Structural models provide a robust theoretical framework to explain the impact of different supply and demand shock as well as policy instruments, but at the same time fail to produce precise NAIRU estimates. A number of empirical studies used this approach to estimate the NAIRU and output gap by removing fluctuations due to demand and supply shocks from the output series. Thus, Funke (1997) proxied demand shocks by manufacturing output and inflation and calculated potential output for Germany by historical decomposition answering the question: what would the output series look like in the absence of demand shocks? In Hjelm (2003) structural VAR approach is used to estimate Swedish NAIRU, output gaps and structural budget balances in the same model. The author concludes that; despite its several weaknesses (long data series requirement and additional data adjustments like, for instance, the integration tests) this method could be useful as a complement to the conventional estimation techniques used by the OECD, EC and other research institutes.

2.3 Combined Methods

Multivariate filters

In response to the criticism of the limitations of the univariate methods, a variety of multivariate extensions of univariate filters have been proposed. Thus, multivariate HP (MVHP) was suggested by Laxton and Teltow (1992) who conditioned the HP filter estimate of cyclical component on additional relevant information. Practically, the residuals of price Phillips curve or/and an Okun's relationship ($\varepsilon^2_{\pi,t}$, $\varepsilon^2_{U,t}$) are added to the classical HP specification (2)[4]:

[4] Cotis et al (2005)

$$HP = \min \left\{ \begin{array}{c} \sum_{t=1}^{T} \left(\ln U_t - \ln U_t^*\right)^2 + \\ +\lambda \sum_{t=1}^{T} \left(\left[\ln U_{t+1}^* - \ln U_t^*\right] - \left[\ln U_t^* - \ln U_{t-1}^*\right]\right)^2 + \\ + \sum_{t=1}^{T} \beta_t \varepsilon_{\pi,t}^2 + \sum_{t=1}^{T} \gamma \varepsilon_{U,t}^2 \end{array} \right\} \tag{10}$$

Another example is a multivariate Kalman filter, which is an extension of the univariate case by taking into account additional equations, for example Phillips curve and/or Okun's law.

Kuttner's unobserved components model (Watson-Clark-Kuttner model)

Watson-Clark unobserved component model makes assumption about the stochastic properties of the observed series and decomposes it into a trend and a cyclical components. If no other information is taken into account, the approach reduces to a purely statistical filter and interpreting the two components as potential (permanent) unemployment and transitory unemployment is entirely speculative. Its advantage, however, consists in the fact that the filter allows for incorporating the structural information. Thus, Watson-Clark-Kuttner model (also called a Kuttner unobserved component model or bivariate stochastic model) is an approach used to estimate NAIRU by combining statistical modelling (Unobserved component method based on univariate Kalman filter) and some elements of economic structure (i.e. a Phillips curve equation), thus linking cyclical component of observed time series to inflation. Kuttner complemented Watson-Clark unobserved component model (3-6) with an equation that relates cycle and changes in inflation rate:

$$\Delta\pi_t = \mu_\pi + \gamma\Delta y_{t-1} + \beta_1 c_{t-1} + \theta(L)\alpha_{\pi t} \tag{11}$$

where $\Delta = 1 - L$, μ_π is constant, $\theta(L)$ is a moving average of order q (i.e. $\theta(L) = 1+ \theta_1 L + ...+ \theta_q L^q$) and $\alpha_{\pi t}$ is a Gaussian white noise variance V_π.

System of equations (3)-(6) complemented by (11) is a bivariate model which can be represented in a state-space format and estimated using Kalman filter and maximum likelihood.[5]

The model application was firstly proposed by Kuttner for the US economy; it was later used to estimate the output gap for the G7 countries and for the EMU area.

An illustration of using alternative Kalman filter and MVHP for estimating NAIRU for the 21 OECD countries is provided in OECD (2000). Trends in two sets of estimates are found to be broadly similar , while MVHP estimates tend to be more closely centered around actual unemployment and Kalman filter estimates tend to be lower and suggest larger degrees of excess supply (especially for the European countries in the early to mid-1980s and mid- to late 1990s). Later made authors conclude the Kalman filter estimates to be the preferred as being economically more appropriate.

Apel-Jansson model

[5] See Durbin and Koopman, 2001, pp.115-118 for the algorithm description)

While production function approach makes use of inflation information to estimate the level of NAIRU and then exogenously inserts NAIRU into a (Cobb-Douglas) production function equation together with other input components, it does not account for the mutual dependence between unemployment gap and output gap. Apel and Jansen (1999) proposed an alternative approach for estimation potential output and the NAIRU, which extends Kuttner's model by additionally taking into account mutual dependence between cyclical unemployment and cyclical output.

Identification is achieved by using a Phillips curve and Okun's law relation.

"Triangle" Phillips model was specified by Gordon (1997) and determines inflation by three factors: expectations/inertia, the pressure of demand as proxied by unemployment and supply shocks.

$$\Delta\pi_t = \sum_{i=1}^{3} p_i \Delta\pi_{i-1} + \sum_{j=0}^{1} \eta_j (u_{t-j} - u_{t-j}^n) + \sum_{k=0}^{4} w_k z_{t-k} + \varepsilon^{PC} \tag{12}$$

with π_t log difference of CPI, u_t the unemployment rate, u_t^n the NAIRU, z_t supply shock proxies. According to the model specification, the estimated position of the NAIRU depends on the development of actual inflation.

Okun's law relation associates cyclical unemployment fluctuations with cyclical output movements:

$$y_t - y_t^p = \sum_{l=o}^{1} \phi_l (u_{t-l} - u_{t-l}^n) + \varepsilon_t^{OL} \tag{13}$$

The NAIRU is assumed to follow a random walk and potential output is assumed to fallow a random walk with drift:

$$u_t^n = u_{t-1}^n + \varepsilon^N \tag{14}$$

$$y_t^p = \alpha + y_{t-1}^p + \varepsilon^P \tag{15}$$

Though the random walk is a standard assumption in such types of models, other processes are also feasible within this framework and one can incorporate possible structural determinants of potential output and NAIRU.

Evolution of cyclical unemployment is assumed to be a purely autoregressive process:

$$u_t - u_t^p = \sum_{m=1}^{2} \delta_m (u_{t-m} - u_{t-m}^n) + \varepsilon_t^C \tag{16}$$

It is convenient to rewrite this model in the state space form, where unknown parameter and unobserved components are estimated by applying Kalman filter and maximum likelihood.

Combined methods are widely used for estimation of NAIRU and output gap since the seminal contribution of Kuttner (1994) who put a bivariate stochastic model on the US data. Apel and Jansson (1999) applied this methodology to Sweden, the UK, the US and Canada. In 2000 OECD has taken up this approach for estimation of NAIRU's for different countries. The use of the Kalman filter to estimate the NAIRU follows a proliferation of recent studies including Gordon (1997 and 1998), King et al. (1995), Staiger et al. (1997a) where it is applied to the United States, Bank of England (1999) to the United Kingdom, Gruen et al. (1999) to Australia, Irac (1999) to France, Meyler (1999) to Ireland, Apel and Jansson (1998, 1999) to Sweden, Rasi and Viikari (1998) to Finland, Orlani and Pichelman (2000) for the European Union and Fabiani and Mestre (1999) to the euro area. There are fewer studies where the approach is applied consistently across a number of countries, although Laxton et al. (1998b) and Laubach (1999) both apply it to all the G7 countries.

3 Estimation procedure

3.1 Empirical framework

In the estimation work reported here we used a combined approach built on mingling of the production function approach with the statistical filtering. Based on the review of alternative theoretical approaches (reduced variant of which is presented in the previous Section), a combined production function model is considered to be the most appropriate framework for estimating NAIRU and potential labour output. We suggest to base the model on the Phillips-type relationship and estimate potential employment using two structural equations: wage setting equation in the form of expectation augmented Phillips curve and an employment equation allowing for supply shocks as proxied by labour force participation rate.

Following the approach used by the EC to estimate potential output of the member states we define sustainable rate of labour utilization (N_t^*) using the following structural equation:

$$N_t^* = HRS_t^* \times PWA_t \times PR_t^{HP*}(1 - U_t^*) \tag{17}$$

where HRS_t^* is an index of trend hours worked, PWA_t is the population of working age, PR_t^{HP*} is the trend participation rate and U_t^* is the trend unemployment rate (NAIRU). All the star-variables are assumed to be endogenous.

A labour input, according to the methodology, is decomposed into the number of employees and the average hours worked per employee which provides a more meaningful measure for the rate of technological progress. In many previous estimation exercises the TFP was biased downwards due to the secular decline in the average hours worked per employee. Introduction of hours worked affects how the potential growth is attributed to the various factors of production, especially labour and TFP, with TFP in general being increased and labour being correspondingly reduced. The HRS_t^* series is smoothed using an ARIMA process.

The estimation procedure of the potential employment is reduced to determining the trend of labour input and is performed in several steps. Definition of trend input of labour is started from the maximum possible level, namely the population of working age (PWA_t). From here we calculate the trend labour force by mechanically detrending (using HP filter) the participation rate (PR_t).

$$LF_t = PR_t^{HP} \times PWA_t \tag{18}$$

In a next step we calculate trend unemployment to be consistent with the NAIRU. The NAIRU is estimated from the modified Phillips curve equation ("triangle" Phillips model) using the Kalman filter. A specialized GAP program was developed by C.Planas and A.Rossi at the Joint Research Center (a division of the EC) at Ispra, Italy, and provides a convenient interface for specifying and estimating Watson-Clark-Kuttner model. Program GAP was used in the current estimation exercise to compute the unobserved NAIRU time series for different regions in Belgium and at the national level.

The idea behind Kalman filtering when applying to the estimation of the NAIRU is essentially the following. The observed unemployment rate (U_t) is decomposed into a trend (U_t^T) and cyclical (U_t^C) components:

$$U_t = U_t^T + U_t^C \tag{19}$$

Both components are treated differently

- U_t^T (NAIRU) is estimated by the time-series model, which captures its general statistical properties (like, for example, non stationarity of the structural unemployment). Economic information which can explain structural unemployment is regarded as unobservable.
- U_t^C (unemployment gap, i.e. observed unemployment minus NAIRU) is modelled with the use of economic information: the link between changes in inflation and cyclical unemployment derived from the Phillips curve.

Trend component of unemployment is assumed to have a stochastic trend[6] and is modelled as a random walk with drift :

$$U_t^T = \mu_t + U_{t-1}^T + z_t \tag{20}$$

Most empirical studies assume no systematic trend in the NAIRU and impose $\mu_t = 0$[7].

In order to estimate the *cyclical* component of unemployment we rewrite the Phillips curve equation and add some exogenous variables:

$$\Delta\pi_t = \alpha + \gamma X_t + \beta U_t^C + e_t \tag{21}$$

where $\Delta\pi_t$ is a change in wage inflation, $U_t^C = (u_t - nairu_t)$ is unemployment gap, X_t is a vector which contains exogenous variables such as wage inflation, changes in the participation rate and work mobility between regions; other unobserved shocks are captured by the error term e_t.

[6] It is more appropriate to model economic time series as having stochastic rather then deterministic trend because high predictability implied by deterministic trend is hardy associated with economic forces that can cause shifts in unemployment. Large unpredictability assumes a random component in the model.

[7] Fabiani and Mestre (2000), Staiger et al (1996), Gordon (1996).

After having estimated the NAIRU, we can calculate potential employment and correct our estimates for the trend in hours worked.

3.2 Data

Data used for this project comes from different sources. While some time series on hand go back to 1980, data on other variables was put available only from the later period. As a result, the estimation period when all available series have common time characteristics, covers 22 years starting from 1986 up to 2007. Observed harmonized rate of unemployment at the national level (for the whole estimation interval) and at the regional level (for the recent years, 1999-2007) was obtained from the National socioeconomic database of the National Bank of Belgium (Belgostat); older regional series came from the National Institute of Statistics.

Data on population of working age, employment and unemployment (used to calculate the participation rate) comes from the Belgian Labour Force Survey and was obtained from the National Institute of Statistics. Labour force participation rate plays a central role in the study of the size and dynamics of country's human resources and in making projections of the potential supply of labour. The derived information can also be used to formulate employment policies and rates of accession to (and retirement from) economic activity, which is crucial for national and regional financial planning of social security systems[8].

Time series of the labour mobility between regions were provided by the National Institute of Statistics. It was decided to incorporate this indicator as an exogenous variable into the structural equation used to estimate the NAIRU since work commuters flows became an unambiguous characteristic of the Belgian labour market in the recent years.

Changes in inflation (in the expectations-augmenting modified Phillips curve) were measured by the wage inflation instead of CPI. Besides simple reasoning built on the higher significance of the wage (comparing to the CPI) considering our research objectives, the decision to use this economic indicator for estimating NAIRU was also motivated by the fact that, under assumption of rationality of labour market participants and considering the official yearly indexation of labour income, wage setting fully incorporate real inflation in Belgium.

Since the data on wage inflation at the regional level is not available, we used national time series for all three regional models. It can be argued, that such wage costs equalization between different regions might have caused certain bias in the estimation results. However, such inexactness of the regional wage inflation estimates can be considered insignificant since official wage settings (such as, for example, minimum wage and unemployment benefits rate) are fixed within the country while main divergence between regional labour markets consist in the different employment opportunities and job structure.

[8] Key Indicators of the Labour Market (KILM) 5th Edition software http://www.ilo.org/public/english/employment/stratkilm/download/kilm01.pdf

3.3 Estimation results

The NAIRU estimation is performed by means of the GAP program, which allows to compute an unobserved component of historical time series and to forecast the future behavior of NAIRU dependent on the exogenously imposed shocks. Cyclical component of unemployment is estimated via the Phillips curve modified equation (see Annex General specification of the model used for the NAIRU estimation for the detailed explanation) which incorporates changes in participation rate and work mobility (commuting) between regions as exogenous variables.

While there is no much doubts about casual link between participation rate and NAIRU dynamics, inclusion of commuting into the equation requires an additional explanation. High work mobility is a specific feature of the Belgian labour market with especially large flows of commuters between Brussels and Flanders. Figures in Table 1 witness a substantial number of interregional work commuters. Thus, the biggest home-work flows in 2007[9] were observed in the following directions: Brussels-Flanders (10.46%), Flanders-Brussels (8.74%) and Wallonia-Brussels (9.44%). Inclusion of the work mobility as an exogenous variable into the second equation helps to achieve a better fit of the model to the regional labour markets behavior. Besides, estimation of the significance of the coefficient of the regional mobility rate provides a valuable policy implication.

	place of work:								
	Brussels		Flanders		Wallonia		Abroad		Total
	1000s	%	1000s	%	1000s	%	1000s	%	1000s
residence:									
Brussels	321	84.00	40	10.46	16	4.29	5	1.25	383
Flanders	236	8.74	2386	88.47	24	0.90	51	1.89	2697
Wallonia	123	9.44	40	3.08	1089	83.71	49	3.78	1300

Table 1: Work mobility in Belgium, 2007

Work mobility was incorporated into the Phillips curve equation by means of two different coefficients alternatively: mobility rate and a share of "in" commuters of the total number of active jobs in the region. Mobility rate coefficients were provided by the NIS and show the share of mobile workers, who work outside their region of residence, in the total number of employed in regions and in the country as a whole. On order to provide an insight view on the principal directions and nature of the labor mobility between different Belgian regions we constructed another measurement of the rate of commuting - a share of "in" commuters in the total number of active jobs in the region. Dynamics of both coefficients is presented in Figure 1a,b. It can be clearly seen from Figure 1b, that main inflows of commuters take place in Brussels.

[9] as a sare of the total working population of the region

Figure 1. Labor mobility across regions, 1986-2007

While the classical supply-oriented explanation of work migration (workers move towards more prosperous regions) should not be neglected, we however consider the suburbanization to be more important in explaining such impressively high inflows of commuters towards Brussels. More explicitly, a rising welfare has created the possibility for more workers to buy family dwellings and move out from Brussels. Accompanied by the growing efficiency and affordability of transport infrastructure, this process considerably reshaped the population composition of Brussels and surrounding Flemish areas: many high-skilled workers has chosen to reside outside Brussels and spend more time on travelling to their work places, while low-skilled workers and low-income ethnic minorities stayed in the capital. This can explain the high level of unemployment rate observed in Brussels, accompanied by the highest level of commuters inflows among all Belgian regions.

Two estimation sets have been obtained using alternative measurement of the working migration, as explained above. The NAIRU in both estimation sets do not show essential dissimilarities. The only disparity is that when work mobility is measured by the share of in-commuter in the total number of active jobs, the NAIRU estimates tend to be more closely centered around actual unemployment, which is quite intuitive since inflows of commuters are more closely related to the unemployment rate in the region then out-commuters' migration dynamics.

The estimation output of the model for different regions and for Belgium as a whole is presented in Table 2 and Figure 2. As it can be clearly seen from the graphs at Figure 2, NAIRU dynamics differs across the regions. The effect of high NAIRU in Wallonia and high and growing NAIRU in Brussels region is partly offset by the comparatively low and decreasing NAIRU in Flanders, which as a result provide stability of NAIRU time series at the national level. Model provides a good fit to the national and regional data on actual unemployment (R-squared is above 90 per cent in all the cases) and our results for a country as a whole look pretty similar to the ones obtained by the EC (see EC, 2006, Reference Manual for the ECFIN's Production Function Technology, p. 17).

	Belgium			Brussels		
	coeff-t	SE	t-stat	coefficient	SE	t-stat
lag wage inflation	0.1796	0.1852	0.9698	0.3639	0.2221	1.6390
participation rate	-0.0146	0.0436	-0.3359	-0.1697	0.0989	-1.7150
mobility rate	-0.5455	0.3362	-1.6222	0.1685	0.1313	1.2833
R-squared	0.9289			0.9297		
	Flanders			Wallonia		
lag wage inflation	0.2521	0.1797	1.4027	0.2000	0.1691	1.1832
participation rate	-0.0525	0.0433	-1.2128	0.0821	0.0761	1.0780
mobility rate	-0.2466	0.3223	-0.7650	-0.5676	0.2434	-2.3318
R-squared	0.9277			0.9360		
Period	1986-2007					
No of obs.	22					

Table 2: Estimation output

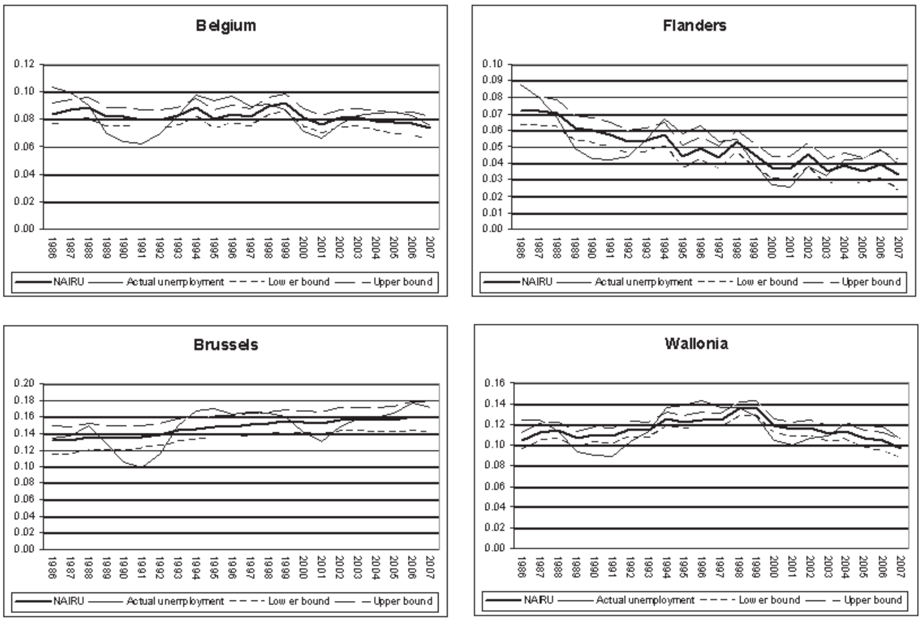

Figure 2. NAIRU versus actual unemployment.

Since the most obvious reason the labor force changes is demographics, we further extended our estimation sample in order to check for the age-specific changes in the NAIRU. We have considered three cohorts including male and female workers aged 15-24 (teenagers and young adults), 25-49 (prime age adults) and 50-64 years (older adults) and estimated cohort-specific NAIRUs for each region and for the whole country. As a result we got quite different estimates for the regional NAIRU within each cohort, with the highest NAIRU being observed in Brussels and the lowest in Flanders.

Thus, if we look at the youngest cohort, we can see that the national NAIRU is slightly increasing, showing the dynamics which is different from the 15-64 years sample. Recall from the previous estimation results that the national NAIRU shows an upward sloping trend with high and high and increasing NAIRU in Wallonia and Brussels respectively and comparatively low and decreasing NAIRU in Flanders. Now, instead, we get almost constant NAIRU in Flanders and increasing NAIRU in other two regions, with Brussels estimates showing three major tweaks - in 1991, 1999 and 2002. Growth of the NAIRU in Brussels in 1990s can be explained by the changing age structure as the baby boom generation has moved through the labour force and by second-wave of migration, when younger cohorts followed up their parents once the later have settled and made initial living and work arrangements.

For the prime-age adults the model appears to have captured the general trends and even the point estimates show persistent similarity. For age cohorts between 50 and 64 years the NAIRU for Brussels and Walloon region show steady resemblance with a growing trend from 1991 till late 90s followed by a decrease till 2000, and a persistent growth till 2007.

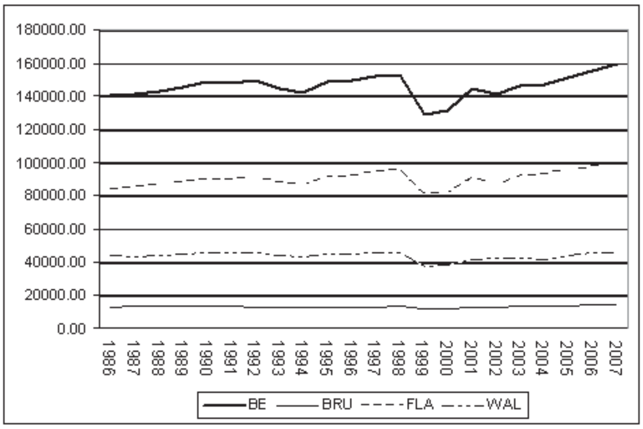

Figure 3. Potential employment

Having estimated NAIRU, we calculated potential employment, using equation (17). After applying HP filtering to detrend participation rate and average hours worked time series, we calculate the potential employment consistent with the NAIRU. Determinants of labour potential are presented in Table 3 (in Appendix), while the dynamics of potential employment for Belgium and for three regions is shown in Figures 3 and 4 (in Appendix). Our results suggest that potential employment at the national level shows an upwards sloped trend, driven mostly by the increasing employment in Flanders. There are no substantial changes in the labour markets potential both in Brussels and Wallonia with the later showing a slightly increasing potential employment from 2001 onwards.

A General specification of the state space model

State space model is a way to subsume a whole class of special cases of interest in time series (in much the same way that linear regression does). Was firstly introduced in Kalman (1960) and Kalman and Bucy (1961), was originally used in aerospace-related research and later largely applied to modeling data in economics.

The model employs a first order autoregression vector as the state equation, which determines the rule for the generation of the unobserved state vector x_t with dimension $p \times 1$ from the past state vector x_t with the same dimension:

$$x_t = \Phi x_{t-1} + w_t \tag{22}$$

It is assumed that w_t is $p \times 1$ dimensioned, independent and identically distributed with zero mean and covariance matrix Q.

Since we do not observe the state vector x_t directly but through its linear transformation with noise added, state space model adds an observation equation of the form

$$y_t = A_t x_t + v_t \tag{23}$$

where A is a $q \times p$ observation matrix. Output vector (or vector of observations) y_t has a dimension $q \times 1$ and contains an observed data. Observed series can be larger or smaller then p, the dimension of the underlying series of interest. Noise v_t is assumed to be white and Gaussian with $q \times q$ covariance matrix R. w_t and v_t are assumed to be uncorrelated.

An underlying idea is that the development of the system over time is determined by x_t according to (22) The basic model can be extended by including exogenous variables into the state or into the observation equations. For example, if we assume to have a $r \times 1$ vector of inputs u_t , then the state space model takes the form of:

$$\begin{aligned} x_t &= \Phi x_{t-1} + \Psi u_t + w_t \\ y_t &= A_t x_t + \Gamma u_t + v_t \end{aligned} \tag{24}$$

with $p \times r$ dimension Ψ and $q \times r$ dimension Γ.

An advantage of this approach is related to the possibility to

· estimate the unknown parameters contained in A_t, Φ, Ψ, Q and R, that define the particular model, and

· estimate or forecast values of the underlying unobserved process x_t .

We can treat missing data configurations and generate a vast spectre of additional models. Another advantage lies in the analogy between the observation matrix A_t and the design matrix in usual regression allows to generate fixed and random effect structures (either constant or time-varying) simply by making appropriate choice for the matrix A_t and the transition structure Φ.

In practice the state space models as defined by (22) and (23) or (24) are used to estimate the underlying unobserved variable x_t for time s given the data $Y_s = \{y_1, ..., y_s\}$.

When $s < t$, the problem is called *forecasting (or prediction).*

When $s = t$, the problem is called *filtering.*

When $s > t$, the problem is called *smoothing.*[10]

In brief, Kalman filter generates, for a given set of model parameters and starting values, a sequence of optimal conditional predictions of the observable variables. The prediction errors are then used in a maximum likelihood routine to find the optimal set of parameters and the corresponding estimates of unobserved parameters.

[10] All abovementioned probelms can be solved using Kalman Filter and Kalman Smoother.

B General specification of the model used for the NAIRU estimation.

1st equation

Is specified similarly to regression models with ARIMA errors [11]

$$U_{1t} = \sum_{i=1}^{M_1} \alpha_{1i} Z_{1it} + \widetilde{U_{1t}} \tag{25}$$

where Z_{1it} is a vector of $M_1 \leq 10$ exogenous variables. The reminder of this regression, $\widetilde{U_{1t}}$ is described as made up of a long term component (trend), U_{1t}^T, and of a short term (cyclical) component U_{1t}^C according to:

$$\widetilde{U_{1t}} = U_{1t}^T + U_{1t}^C \tag{26}$$

Trend component (NAIRU) is assumed to have a stochastic trend and is modelled as a random walk with a zero drift:

$$U_{1t}^T = U_{1t-1}^T + v_t^T \tag{27}$$

Cyclical component is AR(2):

$$(1 - \phi_1 L - \phi_2 L^2) U_{1t}^C = v_t^C \tag{28}$$

v_t^T and v_t^C are permanent and transitory shocks, independent Gaussian white noises (white shock innovations) with variances V_T and V_C respectively.

When exogenous variables are used, they are then assigned to the trend component U_{1t}^T so that the final decomposition is:

$$U_{1t} = U_{1t}^{TF} + U_{1t}^C \tag{29}$$

where the final trend U_{1t}^{TF} is such that:

$$U_{1t}^{TF} = \sum_{i=1}^{M_1} \alpha_{1i} Z_{1it} + U_{1t}^T \tag{30}$$

Putting it all together:

$$U_{1t} = \sum_{i=1}^{M_1} \alpha_{1i} Z_{1it} + U_{1t}^T + U_{1t}^C \tag{31}$$

2nd equation

The model is complemented (following Kuttner) with an additional transition equation that incorporates an economic structure and relates the cyclical component (change in unemployment) U_{1t}^C and inflation. In other words, series 1 is related to stationary transformation of the second series, i.e. change in inflation:

[11] See Planas Ch. and Rossi A. Program GAP, Version 3.1. Technical Description and User-Manula, February 2004, p.4 for more detailed specifications

$$\Delta\pi_{2t} = \alpha_{2\pi} + \sum_{i=1}^{M} \gamma_i X_{2it} + \beta_1 U^C_{1t-1} + \theta(L) v^\pi_t \tag{32}$$

where $\Delta = 1 - L$, $\alpha_{2\pi}$ is a constant, $\theta(L)$ is a moving average of order q, i.e. $\theta(L) = 1 + \theta_1 L + ... + \theta_q L^q$.

X_{2it} is a vector of M exogenous variables

C Tables and Figures

year	growth of the PWA				trend PR (%)				NAIRU			
	BE	BRU	FLA	WAL	BE	BRU	FLA	WAL	BE	BRU	FLA	WAL
1986	x	x	x	x	58	57	59	56	8.43	13.35	7.15	10.36
1987	1.3	0.2	1.7	-0.2	58	58	59	56	8.68	13.31	7.15	11.16
1988	0.0	0.3	0.1	0.6	58	58	59	57	8.91	13.73	7.01	11.31
1989	0.3	-0.3	0.3	0.0	59	58	60	57	9.16	13.60	6.12	10.49
1990	-0.1	-0.8	0.0	-0.1	59	59	60	58	8.19	13.62	5.99	10.99
1991	-0.1	-1.4	0.1	0.3	60	60	60	58	7.96	13.73	5.74	10.91
1992	0.2	-0.4	0.2	0.2	60	60	61	59	7.95	13.91	5.30	11.60
1993	0.2	-0.1	0.3	0.1	61	61	61	59	8.22	14.38	5.43	11.52
1994	0.1	-0.1	0.2	0.0	61	61	62	60	8.84	14.52	5.76	12.61
1995	0.1	-0.3	0.2	-0.2	62	62	63	60	7.99	14.70	4.43	12.19
1996	0.0	0.3	0.0	0.0	62	62	63	60	8.38	14.73	4.89	12.46
1997	0.1	0.4	0.1	0.0	63	62	64	61	8.15	15.03	4.37	12.33
1998	0.0	0.4	0.0	0.0	63	63	64	61	8.96	15.13	5.27	13.37
1999	0.1	0.2	0.0	0.1	64	63	65	61	9.24	15.37	4.56	13.47
2000	0.1	0.7	0.0	0.2	64	63	66	62	8.12	15.27	3.73	11.84
2001	0.1	0.6	0.0	0.2	65	64	66	62	7.62	15.19	3.68	11.49
2002	0.4	1.7	0.2	0.5	65	64	67	62	8.06	15.56	4.52	11.68
2003	0.5	1.9	0.3	0.5	65	64	67	63	8.09	15.65	3.49	11.11
2004	0.4	1.0	0.2	0.6	66	65	68	63	7.82	15.61	3.83	11.41
2005	0.8	1.5	0.6	1.1	66	65	68	63	7.68	15.66	3.56	10.71
2006	0.9	1.6	0.8	1.0	67	65	69	63	7.61	15.91	3.93	10.45
2007	1.0	1.5	0.8	1.0	67	66	69	64	7.27	15.84	3.30	9.67

Table 3: Determinants of labour potential

year	BE	BRU	FLA	WAL
1986	140.22	12.85	83.64	43.68
1987	141.90	13.10	84.92	43.32
1988	143.03	13.06	86.35	43.75
1989	144.93	13.08	88.70	44.68
1990	148.52	13.39	89.44	45.49
1991	147.95	13.00	89.52	45.33
1992	149.41	12.84	91.43	45.02
1993	144.30	12.56	87.56	44.26
1994	142.31	12.49	86.58	43.38
1995	148.67	12.70	91.39	44.77
1996	149.37	12.82	92.43	44.50
1997	152.22	12.75	93.97	45.92
1998	152.89	13.09	95.32	45.04
1999	128.83	11.70	80.72	37.69
2000	130.82	11.66	81.66	38.66
2001	144.40	12.37	90.81	41.94
2002	141.31	12.73	86.77	42.24
2003	146.44	13.09	92.28	42.82
2004	147.11	13.06	92.80	42.08
2005	151.69	13.24	95.62	44.07
2006	155.89	13.74	97.57	45.45
2007	159.90	14.11	100.89	46.25

Table 4: Determinants of labour potential

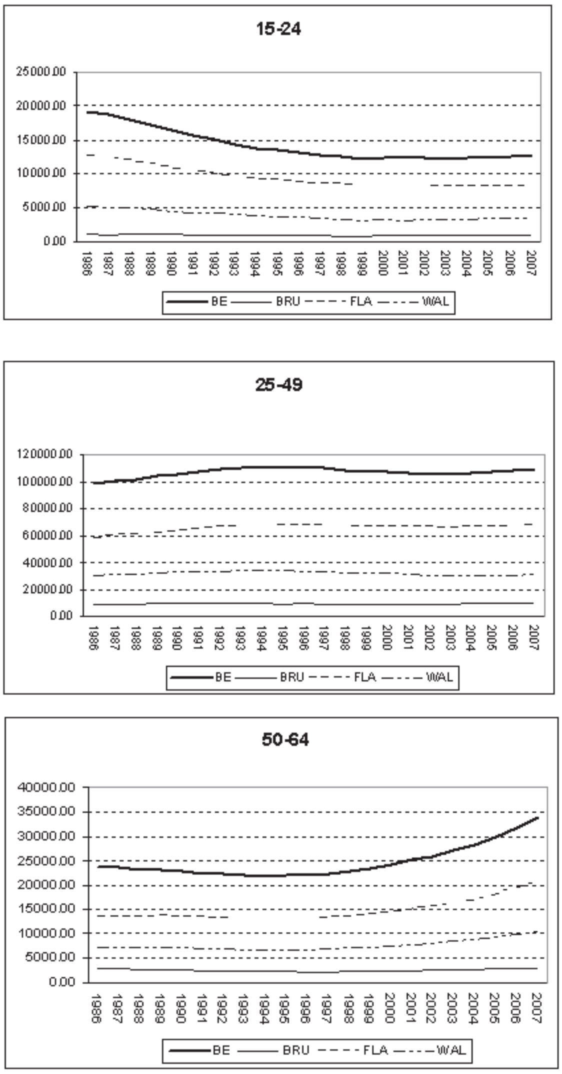

Figure 1: Figure 4. Potential employment for different age cohorts.

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